CS 188: Artificial Intelligence Spring 2009

Lecture 7: Expectimax Search
2/10/2009

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Slides adapted from Dan Klein, Stuart Russell or Andrew Moore

Announcements

- Written Assignment 1:
  - Due at the end of lecture
  - If you haven't done it, but still want some points, come talk to me after class
- Project 1:
  - Most of you did very well
  - We promise not to steal your slip days
  - Come to office hours if you didn't finish & want help
- Project 2:
  - Due a week from tomorrow (Wednesday)
  - Want a partner? Come to the front after lecture

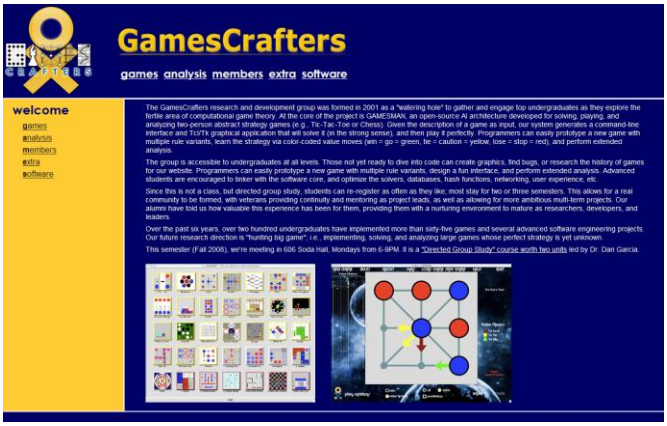
Today

- Mini-contest 1 results
- Pruning game trees
- Chance in game trees

Mini-Contest Winners

- Problem: eat all the food in bigSearch
- Challenge: finding a provably optimal path is very difficult
- Winning solutions (baseline is 350):

GamesCrafters



http://gamescrafters.berkeley.edu/

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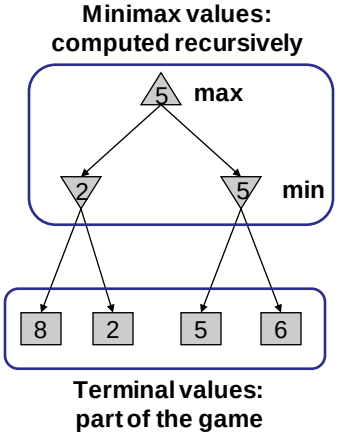
Adversarial Games

- Deterministic, zero-sum games:
  - tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Each node has a minimax value: best achievable utility against a rational adversary



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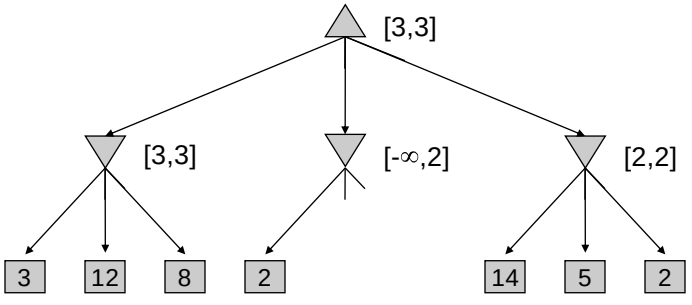
Computing Minimax Values

- Two recursive functions:
  - max-value maxes the values of successors
  - min-value mins the values of successors

def value(state):
    If the state is a terminal state: return the state's utility
    If the next agent is MAX: return max-value(state)
    If the next agent is MIN: return min-value(state)

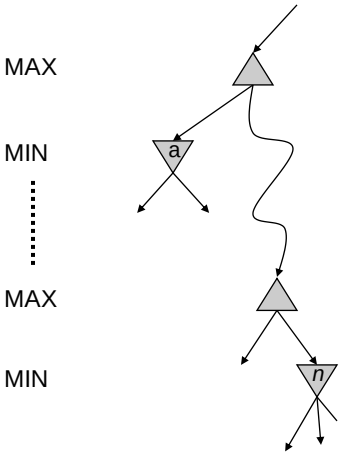
def max-value(state):
    Initialize max = -∞
    For each successor of state:
        Compute value(successor)
        Update max accordingly
    Return max

Pruning in Minimax Search



Alpha-Beta Pruning

- General configuration
  - *a* is the best value that MAX can get at any choice point along the current path
  - If *n* becomes worse than *a*, MAX will avoid it, so can stop considering *n*'s other children
  - Define *b* similarly for MIN



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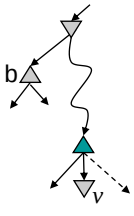
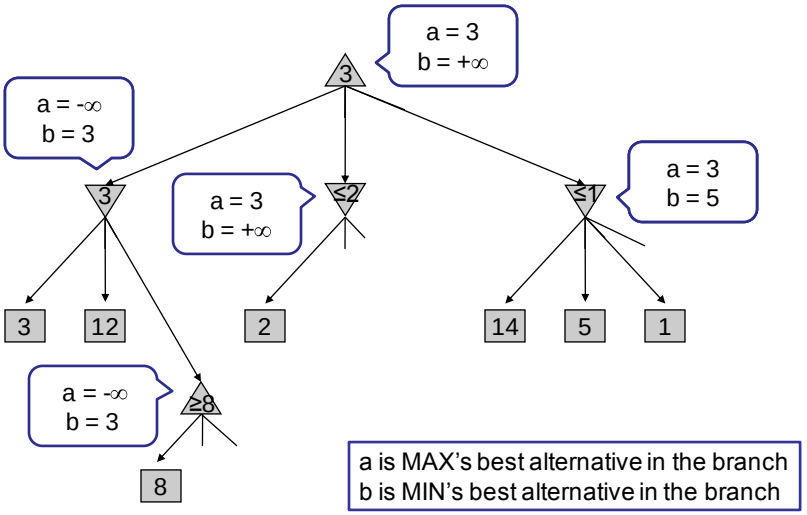
Alpha-Beta Pseudocode

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
  return v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```



Alpha-Beta Pruning Example



Alpha-Beta Pruning Properties

- This pruning has no effect on final result at the root
- Values of intermediate nodes might be wrong!
- Good move ordering improves effectiveness of pruning
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth
  - Full search of, e.g. chess, is still hopeless!
- This is a simple example of metareasoning, and the only one you need to know in detail

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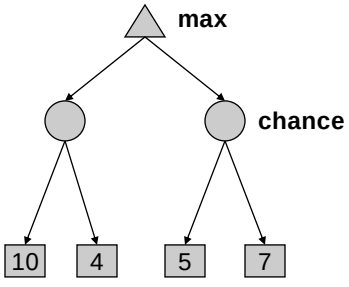
Expectimax Search Trees

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, next card is unknown
  - In monopoly, the dice are random
  - In pacman, the ghosts act randomly
- We can do **expectimax search**
  - Chance nodes are like min nodes, except the outcome is uncertain
  - Calculate expected utilities
  - Max nodes as in minimax search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize the underlying problem as a **Markov Decision Process**



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Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility: an agent should chose the action which maximizes its expected utility, given its knowledge
- General principle for decision making
- Often taken as the definition of rationality
- We'll see this idea over and over in this course!
- Let's decompress this definition…

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Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

- Example: traffic on freeway?
  - Random variable: T = how much traffic is there
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.5, P(T=heavy) = 0.25
  - Common abbreviation: P(light) = 0.5

- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

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What are Probabilities?

- Objectivist / frequentist answer:
  - Averages over repeated *experiments*
  - E.g. empirically estimating P(rain) from historical observation
  - Assertion about how future experiments will go (in the limit)
  - New evidence changes the *reference class*
  - Makes one think of *inherently random* events, like rolling dice

- Subjectivist / Bayesian answer:
  - Degrees of belief about unobserved variables
  - E.g. an agent's belief that it's raining, given the temperature
  - E.g. pacman's belief that the ghost will turn left, given the state
  - Often *learn* probabilities from past experiences (more later)
  - New evidence *updates beliefs* (more later)

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Uncertainty Everywhere

- Not just for games of chance!
  - I'm sick: will I sneeze this minute?
  - Email contains "FREE!": is it spam?
  - Tooth hurts: have cavity?
  - 60 min enough to get to the airport?
  - Robot rotated wheel three times, how far did it advance?
  - Safe to cross street? (Look both ways!)
- Sources of uncertainty in random variables:
  - Inherently random process (dice, etc)
  - Insufficient or weak evidence
  - Ignorance of underlying processes
  - Unmodeled variables
  - The world's just noisy – it doesn't behave according to plan!
- Compare to *fuzzy logic*, which has *degrees of truth*, rather than just *degrees of belief*

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Reminder: Expectations

- We can define function f(X) of a random variable X
- The expected value of a function is its average value, weighted by the probability distribution over inputs
- Example: How long to get to the airport?
  - Length of driving time as a function of traffic:
    L(none) = 20, L(light) = 30, L(heavy) = 60
  - What is my expected driving time?
    - Notation: E[L(T)]
    - Remember, P(T) = {none: 0.25, light: 0.5, heavy: 0.25}
    - E[L(T)] = L(none) * P(none) + L(light) * P(light) + L(heavy) * P(heavy)
    - E[L(T)] = (20 * 0.25) + (30 * 0.5) + (60 * 0.25) = 35

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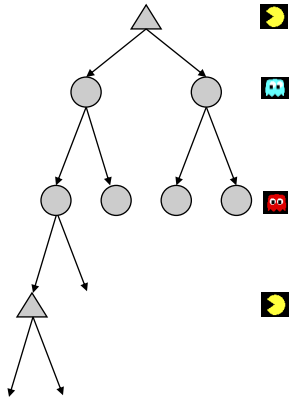
Expectimax Search

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a node for every outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- For now, assume for any state we have a distribution to assign probabilities to opponent actions / environment outcomes



Having a probabilistic belief about an agent's action does not mean that agent is flipping any coins!

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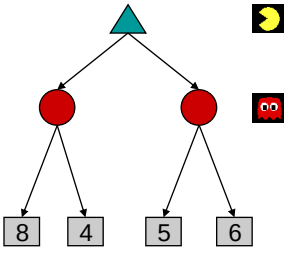
Expectimax Pseudocode

```
def value(s)
    if s is a max node: return maxValue(s)
    if s is an exp node: return expValue(s)
    if s is a terminal node: return evaluation(s)

def maxValue(s)
    values = [value(s') for s' in successors(s)]
    return max(values)

def expValue(s)
    values = [value(s') for s' in successors(s)]
    weights = [probability(s, s') for s' in successors(s)]
    return expectation(values, weights)
```



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Expectimax for Pacman

- Notice that we've gotten away from thinking that the ghosts are trying to minimize pacman's score
- Instead, they are now a part of the environment
- Pacman has a belief (distribution) over how they will act

- Questions:
  - Is minimax a special case of expectimax?
  - What happens if we think ghosts move randomly, but they really do try to minimize Pacman's score?

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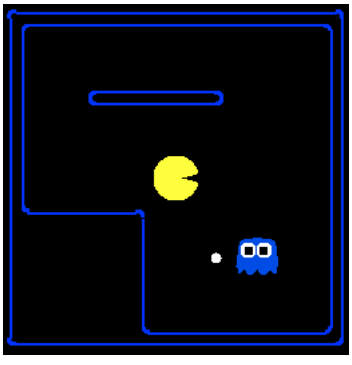
Expectimax for Pacman

Results from playing 5 games

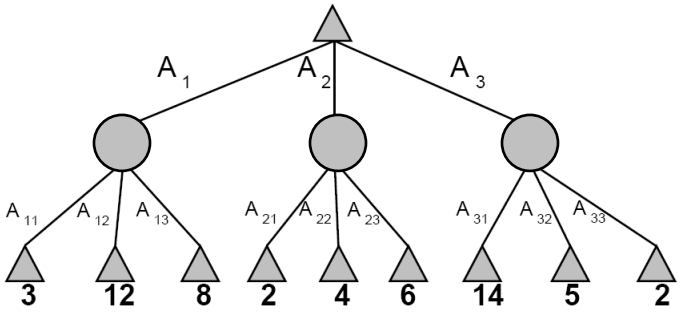
	Minimizing Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 493	Won 5/5 Avg. Score: 483
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503



Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

[Demo]

Expectimax Pruning?



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Expectimax Evaluation

- For minimax search, evaluation function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this property insensitivity to monotonic transformations
- For expectimax, we need the magnitudes to be meaningful as well
  - E.g. must know whether a 50% / 50% lottery between A and B is better than 100% chance of C
  - 100 or -10 vs 0 is different than 10 or -100 vs 0

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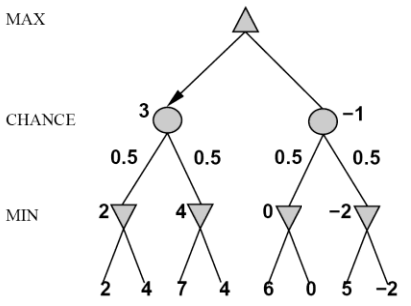
Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax




```
if state is a MAX node then
        return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
        return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
        return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
```

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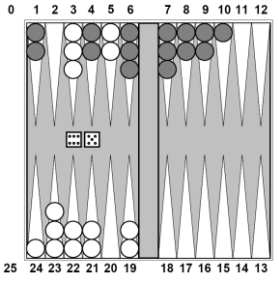
Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play



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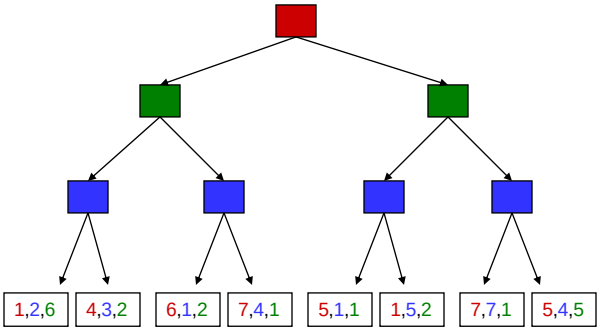
Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children
  - Can give rise to cooperation and competition dynamically…



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